Division by Fractions

This is a story-board, a mock-up of an animation. It can be viewed in Acrobat Reader. Final form will be an animation, perhaps an mpeg file. Most of the text will be spoken (perhaps with text for the hearing impaired, as well.)

TO USE: Use Reader s View menu s Single Page command, then the Page Down key will move to the next frame.

A teacher is welcome to use this story-board in a class (for free) if they
1) send email comments, criticisms and requests to math@strausses.net
2) join the Yahoo razzmath group.
(Send an email to razzmath-subscribe@yahoogroups.com)

More information is available at http://randy.strauss.net/math

First there was a lesson about how multiplication and division are opposites. If we multiply a x b = c, then if we divide c by b, we get a.

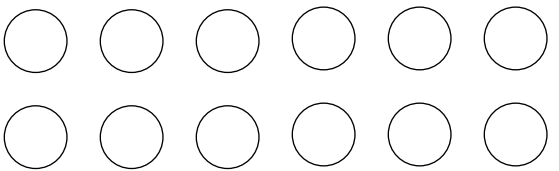
Another lesson (not just an animation) showed how fractions are really division. So multiplying by 1/a is the same as dividing by a.

It s very important that students don t just watch the animations. They need to work with these concepts to get them- and not just work through problems, but divide real sets, and multiply (replicate) real groups.

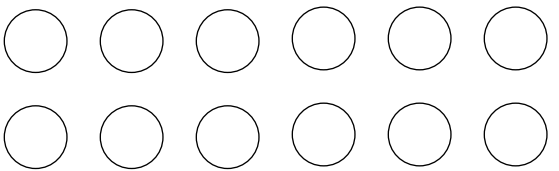
Today s lesson builds on this to show that dividing by a fraction is the same as multiplying by the reciprocal. (The term reciprocal is not introduced.)

(Note that this is an early draft...)

Let s say that I have 12 circles.



If I divide 12 circles into 6 groups,
how many in each group?



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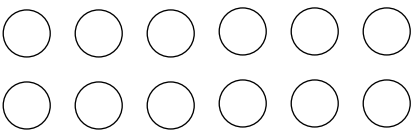
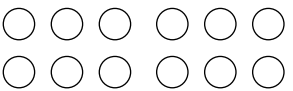
how many in each group?

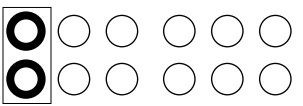


If I divide 12 circles into 6 groups,
how many in each group?



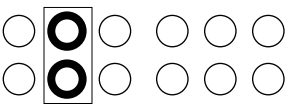
If I divide 12 circles into 6 groups
how many in each group

1 group

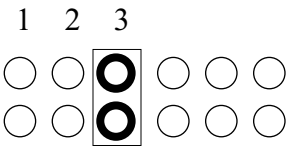


If I divide 12 circles into 6 groups
how many in each group?

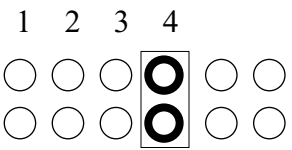
1 2 groups



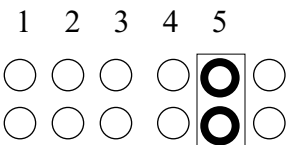
If I divide 12 circles into 6 groups
how many in each group?



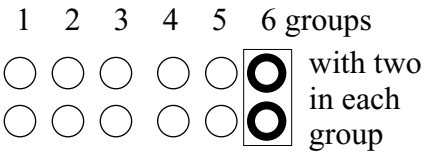
If I divide 12 circles into 6 groups
how many in each group?



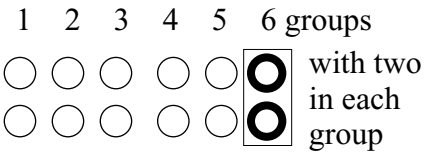
If I divide 12 circles into 6 groups
how many in each group?



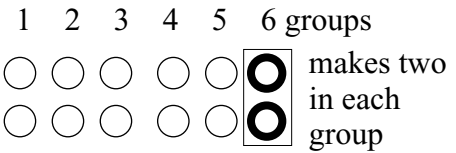
If I divide 12 circles into 6 groups
how many in each group?



Let me say it a bit differently...

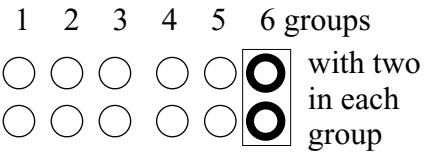


If I make 12 circles be 6 groups
there are 2 in each group.



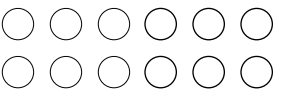
If I make 12 circles be 6 groups
there are 2 in each group?

12 ÷ 6 = 2



If I make 12 circles be **4** groups
there are how many in each group?

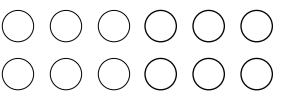
$12 \div 6 = 2$



If I make 12 circles be **4** groups
there are how many in each group?

$12 \div 6 =$ 2

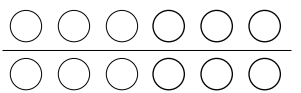
1 group



If I make 12 circles be **4** groups
there are how many in each group?

$12 \div 6 =$ 2

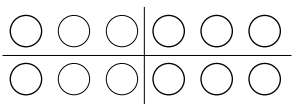
2 groups



If I make 12 circles be 4 groups
there are how many in each group?

$12 \div 6 =$ 2

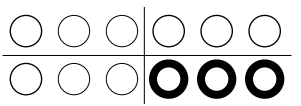
4 groups



If I make 12 circles be 4 groups
there are how many in each group?

$12 \div 6 =$ 2

4 groups

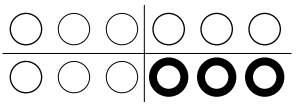


There are 3 in each group.

If I make 12 circles be 4 groups
there are how many in each group?

$12 \div 6 = 2$
$12 \div 4 = 3$

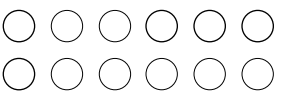
4 groups



There are 3 in each group.

If I make 12 circles be **3** groups
there are how many in each group?

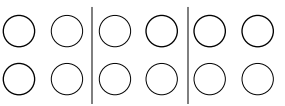
$12 \div 6 = 2$
$12 \div 4 = 3$



If I make 12 circles be 3 groups
there are how many in each group?

$12 \div 6 = 2$
$12 \div 4 = 3$

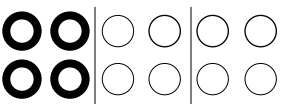
3 groups



If I make 12 circles be 3 groups
there are how many in each group?

$12 \div 6 = 2$
$12 \div 4 = 3$

3 groups

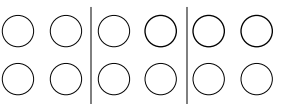


There are 4 in each group.

If I make 12 circles be 3 groups
there are how many in each group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$

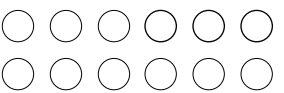
3 groups



There are 4 in each group.

If I make 12 circles be **2** groups
there are how many in each group?

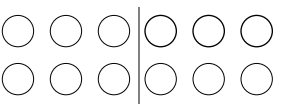
$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$



If I make 12 circles be 2 groups
there are how many in each group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$

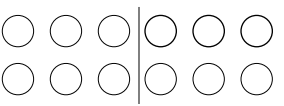
2 groups



If I make 12 circles be 2 groups
there are how many in each group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$

2 groups

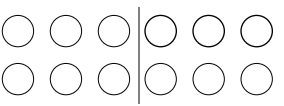


There are 6 in each group.

If I make 12 circles be 2 groups
there are how many in each group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$

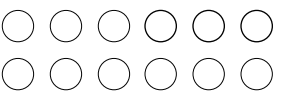
2 groups



There are 6 in each group.

If I make 12 circles be 1 group
there are how many in the group?

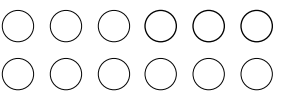
$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$



If I make 12 circles be 1 group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$

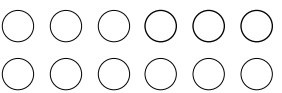
1 group



If I make 12 circles be 1 group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$

1 group

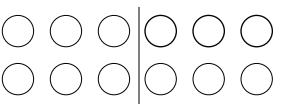


There is 12 in the group.

If I make 12 circles be 1 group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$

1 group

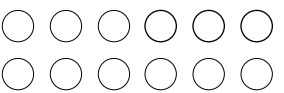


There is 12 in the group.

If I make 12 circles be $\frac{1}{2}$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$

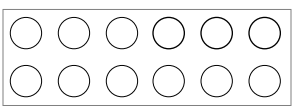
12



If I make 12 circles be $^1/_2$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$

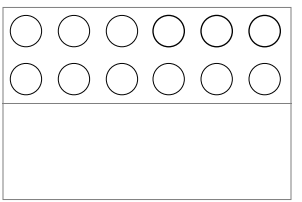
12



If I make 12 circles be $^1/_2$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$

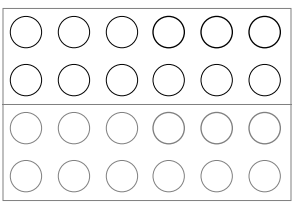
12



If I make 12 circles be $^1/_2$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$

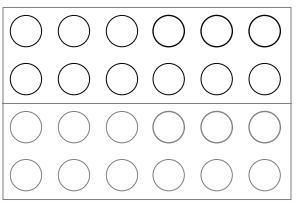
12 is now half of the group.



If I make 12 circles be $^1/_2$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$

12 is now half of the group.

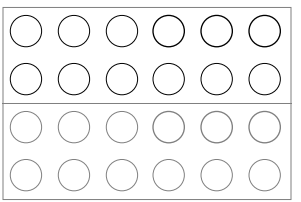


There are 2 x 12

If I make 12 circles be $^1/_2$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$

12 is now half of the group.

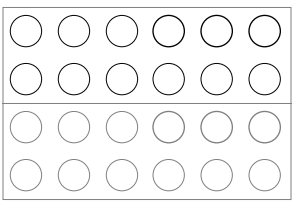


There are 2 x 12 = 24 in the group.

If I make 12 circles be $^1/_2$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$
$12 \div {}^1/_2 = 24$

12 is now half of the group.

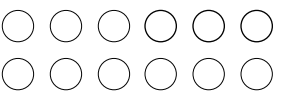


There are 2 x 12 = 24 in the group.

If I make 12 circles be $\mathbf{1/3}$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$
$12 \div {}^{1}/_{2} = 24$

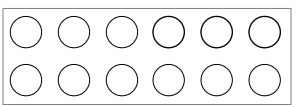
12



If I make 12 circles be $^1/_3$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$
$12 \div {}^1/_2 = 24$

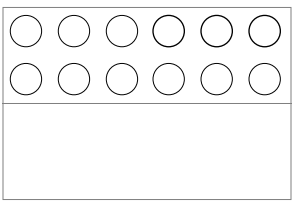
12



If I make 12 circles be $^1/_3$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$
$12 \div {}^1/_2 = 24$

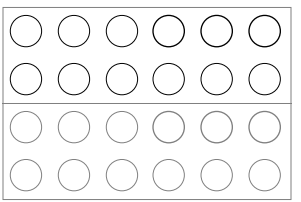
12



If I make 12 circles be $^1/_3$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$
$12 \div {}^1/_2 = 24$

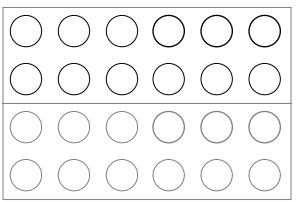
12 is now half of the group.



If I make 12 circles be $^1/_3$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$
$12 \div {}^1/_2 = 24$

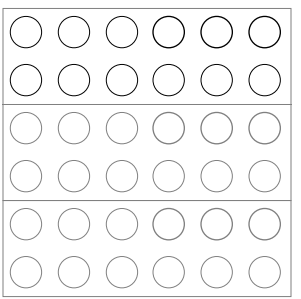
12 is now one **half** of the group.



If I make 12 circles be $^1/_3$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$
$12 \div {}^1/_2 = 24$

12 is now one **third** of the group.



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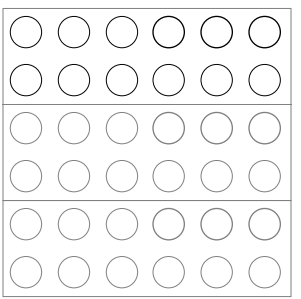
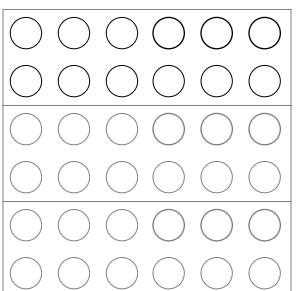


There are 3 x 12 = 36 in the group.

If I make 12 circles be $^1/_3$ of a group
there are how many in the group?

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$
$12 \div {}^1/_2 = 24$
$12 \div {}^1/_3 = 36$

12 is now one **third** of the group.



There are 3 x 12 = 36 in the group.

Dividing by one half

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$
$12 \div {}^{1}/_{2} = 24$ ◄——
$12 \div {}^{1}/_{3} = 36$

Dividing by one half
is the same as multiplying by 2

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$
$12 \div {}^{1}/_{2} = 24$ ◄——— $12 \times 2 = 24$
$12 \div {}^{1}/_{3} = 36$

Dividing by one third

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$
$12 \div {}^{1}/_{2} = 24$ ◄——— 12 x 2 = 24
$12 \div {}^{1}/_{3} = 36$ ◄———

Dividing by one third
is the same as multiplying by 3

$12 \div 6 = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$
$12 \div {}^{1}/_{2} = 24$ ◄——— $12 \times 2 = 24$
$12 \div {}^{1}/_{3} = 36$ ◄——— $12 \times 3 = 36$

Dividing by six

$12 \div 6 = 2$ ◄——

$12 \div 4 = 3$

$12 \div 3 = 4$

$12 \div 2 = 6$

$12 \div 1 = 12$

$12 \div {}^{1}/_{2} = 24$ ◄—— $12 \times 2 = 24$

$12 \div {}^{1}/_{3} = 36$ ◄—— $12 \times 3 = 36$

Dividing by six
is the same as multiplying by 1/6th

$12 \div 6 = 2$ ◄——— $12 \times {}^{1}/_{6} = 2$
$12 \div 4 = 3$
$12 \div 3 = 4$
$12 \div 2 = 6$
$12 \div 1 = 12$
$12 \div {}^{1}/_{2} = 24$ ◄——— $12 \times 2 = 24$
$12 \div {}^{1}/_{3} = 36$ ◄——— $12 \times 3 = 36$

Dividing by x

Dividing by x
is the same as multiplying by 1/x

Dividing by x
is the same as multiplying by 1/x

Dividing by 1/x

Dividing by x
is the same as multiplying by 1/x

Dividing by 1/x
is the same as multiplying by x

The end.

Each animation is designed to be one day s lesson, so each probably seems incomplete.

Today s lesson should be followed by working through other examples and a class discussion and discovery that X / Y is X times 1/Y. The next video will capture this concept and introduce reciprocal.

-Randy